FEATURES OF A LAMINAR FLOW OF VOLATILE BINARY GAS MIXTURES IN PLANE AND COAXIAL CHANNELS

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When a laminar flow of binary gas mixture passes through a plane (Fig. 1) or coaxial (Fig. 2) channel, in which recondensation of the volatile component of the binary gas mixture occurs, the established distributions of mass velocity and pressure in the channel will differ from the velocity and pressure distributions in a gas flow undisturbed by recondensation. A theoretical investigation of flows complicated by recondensation is of interest for practical calculations of heat exchangers, condensers, etc. [1]. In view of this the present investigation was devoted to a study of established laminary isothermal flows of a binary gas mixture in plane and coaxial channels with due consideration of recondensation of molecules of one of the gas mixture components.

The analytical investigations of flows in a channel between coaxial cylindrical surfaces and between two parallel plates are similar and, hence, we will dwell in more detail on the analysis of flow in a coaxial channel.

On one of the surfaces forming the channel absorption of molecules of the first component of the gas, mixture occurs, and on the other surface release (e.g., condensation and evaporation) takes place. We will assume that the relative concentrations c_1 of the first component at the surface of the inner cylinder c_{11} and at the surface of the outer cylinder c_{12} are kept constant

$$c_1|_{\tau=R_1/R_2}=c_{11}=\text{const},\quad c_1|_{\tau=1}=c_{12}=\text{const},$$

where $c_1 = n_1/n$; n is the concentration of gas molecules ($n = n_1 + n_2$); n_1 and n_2 are, respectively, the concentration of molecules of the first and second components of the binary gas mixture; R_1 and R_2 are the radii of the cylinders forming the channel ($R_2 > R_1$); $\tau = r/R_2$; r is the transverse coordinate.

There is no absorption or release of molecules of the second component on the channel boundaries. For an established flow of binary gas mixture in a channel the distributions of the relative concentration c_1 and mass velocity v depend only on r and are characterized [2, 3] by the system of equations

$$\frac{d}{dr}(r\rho v_r)=0; \tag{1}$$

$$\rho v_r\frac{dv_r}{dr}=-\frac{\partial p}{\partial r}-\frac{2}{3}\frac{d}{dr}\left[\frac{\mu}{r}\frac{d}{dr}(rv_r)\right]+\frac{2}{r}\frac{d}{dr}\left(\mu r\frac{dv_r}{dr}\right); \tag{2}$$

$$\rho v_r\frac{dv_z}{dr}=-\frac{\partial p}{\partial z}+\frac{1}{r}\frac{d}{dr}\left(\mu r\frac{dv_z}{dr}\right); \tag{3}$$

$$\frac{d}{dr}(rJ_{1r})=0; \tag{4}$$

$$\frac{d}{dr}(rJ_{2r})=0, \tag{5}$$

where $\rho=m_1n_1+m_2n_2$; m_1 and m_2 are the masses of the molecules of the first and second components; p is the pressure; z is the longitudinal coordinate; $c_2=n_2/n$; $\mathbf{J}_1=n_1\mathbf{v}-D_{12}(n^2m_2/\rho)\nabla c_1$; $\mathbf{J}_2=n_2\mathbf{v}-D_{12}(n^2m_1/\rho)\nabla c_2$; D_{12} is the diffusion coefficient; μ is the dynamic viscosity.

The solution of system (1)-(5) was obtained on condition that on the channel boundaries the conditions

$$v_z|_{\tau=R_1/R_2;1}=0, \tag{6}$$

$$J_{2r}|_{\tau=R_1/R_2;1}=0 \tag{7}$$

are satisfied. The distribution of v_r, v_z, p, c_1, and ρ found in this case have the following form:

$$v_r=\frac{m_1nD_{12}}{R_2\rho\tau}\frac{\ln[(1-c_{12})/(1-c_{11})]}{\ln(R_2/R_1)}; \tag{8}$$

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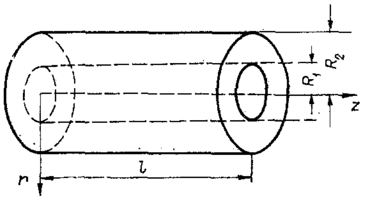

Fig. 1

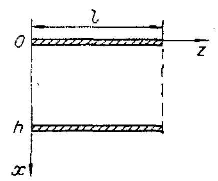

Fig. 2

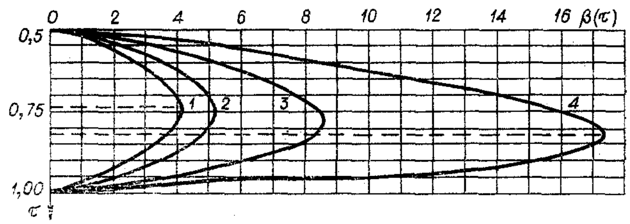

Fig. 3

$$v_z = \frac{\alpha R_2^2}{2}\{\exp[f(\tau)]\}\left\{-\int\limits_{R_1/R_2}^{\tau}\frac{\tau}{\mu}\exp[-f(\tau)]\,d\tau + \int\limits_{R_1/R_2}^{\tau}\frac{\exp[-f(\tau)]}{\mu\tau}\,d\tau\,\frac{\int\limits_{R_1/R_2}^{1}\frac{\tau}{\mu}\exp[-f(\tau)]\,d\tau}{\int\limits_{R_1/R_2}^{1}\frac{\exp[-f(\tau)]}{\mu\tau}\,d\tau}\right\}; \tag{9}$$

$$p = p_0 - \alpha z + \frac{2\mu\eta}{R_2^2}\int\limits_{R_1/R_2}^{\tau}\frac{1}{\tau}\frac{d}{d\tau}\left[\mu\tau\frac{d}{d\tau}\left(\frac{1}{\rho\tau}\right)-\frac{\mu\eta}{2\rho\tau}\right]d\tau + \frac{2}{3}\frac{\mu\eta}{R_2^2}n(m_2-m_1)c_{22}\sigma\left[\frac{\mu}{\rho^2}\tau^{\sigma-2}-\frac{\mu_0}{\rho_0^2}\left(\frac{R_1}{R_2}\right)^{\sigma-2}\right]; \tag{10}$$

$$c_1 = 1-(1-c_{12})\tau^{\sigma}; \tag{11}$$

$$\rho = \{m_1[1-(1-c_{12})\tau^{\sigma}] + m_2(1-c_{12})\tau^{\sigma}\}n. \tag{12}$$

In formulas (8)-(12)

$$p_0 = p|_{\tau=R_1/R_2;\,z=0};\quad \eta = m_1 n D_{12}\sigma/\mu;$$

$$\mu_0 = \mu|_{\tau=R_1/R_2};\quad \rho_0 = \rho|_{\tau=R_1/R_2};$$

$$\sigma = \frac{\ln[(1-c_{11})/(1-c_{12})]}{\ln(R_1/R_2)};\quad f(\tau) = \int\limits_{R_1/R_2}^{\tau}\frac{\eta}{\tau}\,d\tau.$$

It follows from Eqs. (1), (4), and (5) that functions $r\rho v_r$, rJ_{1r}, and rJ_{2r} are constants, independent of r. Through any cylindrical surface of radius $r(R_1 < r < R_2)$ with generatrices of length l in the steady-state case considered here pass equal radial flows of molecules of the first (Q_1) and second (Q_2) components of the gas mixture ($Q_1 = 2\pi r J_{1r} l$; $Q_2 = 2\pi r J_{2r} l$), which accounts for the independence of rJ_{1r} and rJ_{2r} on r.

It follows from condition (7) and the constancy of rJ_{2r} and $J_{2r} = 0$ at any point in the channel. From the expressions for J_{1r} and J_{2r} we obtain $\rho v_r = m_1 J_{1r}$ when $J_{2r} = 0$.

Diffusion of molecules of the first and second components in a transverse direction occurs in the presence of a Stefanov flow of gas mixture. In the case of such diffusion J_{2r} will be zero and $\rho v_r = m_1 J_{1r}$, which agrees with the results that we obtained.

The equality $\rho v_r = m_1 J_{1r}$ is derived also from formula (8), from which it follows that ρv_r is independent of the longitudinal coordinate.

The dynamic viscosity μ in the general case depends on c_1 and, hence, on the transverse coordinate r. If this dependence can be neglected, the distributions of v_z and p will be characterized by the formulas

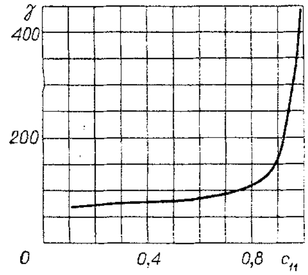

Fig. 4

$$v_z = \frac{\alpha_1 R_2^2}{2\mu(2-\eta)}\left\{1-\tau^2-\frac{\left[1-\left(\frac{R_1}{R_2}\right)^2\right]}{\left[1-\left(\frac{R_1}{R_2}\right)^\eta\right]}(1-\tau^\eta)\right\} = \frac{Q}{2\pi n R_2^2}\beta(\tau); \tag{13}$$

$$p = p_0 - \alpha_1 z + \frac{2\mu^2\eta}{R_2^2}\int\limits_{R_1/R_2}^{\tau}\frac{1}{\tau}\frac{d}{d\tau}\left[\tau\frac{d}{d\tau}\left(\frac{1}{\rho\tau}\right) - \frac{\eta}{2\rho\tau}\right]d\tau + \frac{2}{3}\frac{\mu^2\eta}{R_2^2}n(m_2-m_1)c_{22}\sigma\left[\frac{\tau^{\sigma-2}}{\rho^2} - \frac{1}{\rho_0^2}\left(\frac{R_1}{R_2}\right)^{\sigma-2}\right], \tag{14}$$

where

$$\alpha_1 = Q/(\pi n R_2^2 \Omega);$$

$$\Omega = \frac{R_2^2(1-c_{12})}{(2-\eta)}\left\{\frac{\left[1-(R_1/R_2)^2\right]\left[1-(R_1/R_2)^{2+\eta+\sigma}\right]}{\left[1-(R_1/R_2)^\eta\right](2+\eta+\sigma)} - \frac{\left[(R_1/R_2)^2-(R_1/R_2)^\eta\right]\left[1-(R_1/R_2)^{2+\sigma}\right]}{\left[(R_1/R_2)^\eta-1\right](2+\sigma)} - \frac{1-(R_1/R_2)^{4+\sigma}}{4+\sigma}\right\}.$$

The distributions of v_r and c_1 in the channel are independent of μ and are given by formulas (8), (11). Formulas (13), (14) show that the distributions of v_z and p in the gas mixture flow for a given flow rate Q of the nonvolatile component depend on the drop of concentration of the volatile component in the channel, i.e., on c_{11} and c_{12}.

To illustrate the dependence of v_z on τ, c_{11}, and c_{12}, Fig. 3 shows plots of the variable $\beta(\tau)$ against τ

$$\beta(\tau) = \frac{R_2^2}{\mu(2-\eta)\Omega}\left\{1-\tau^2-\frac{\left[1-(R_1/R_2)^2\right](1-\tau^\eta)}{\left[1-(R_1/R_2)^\eta\right]}\right\}$$

for a vapor–air mixture with temperature 293°K, pressure p_0 = 1 atm, $\mu = 2\cdot10^{-5}$ kg/m·sec, c_{12} = 0, and different values of c_{11} (curve 1 corresponds to c_{11} = 0, 2 to c_{11} = 0.5, 3 to c_{11} = 0.9, 4 to c_{11} = 0.995). The calculations were made for a channel with R_1/R_2 = 0.5. As Fig. 3 shows, an increase in c_{11} for a prescribed flow rate Q leads to an increase in the maximum value of v_z and shift of the point τ_{max}, at which v_z is maximal, towards the outer cylinder. The increase in the longitudinal velocity component with increase in c_{11} (see Fig. 3) is due to an increase in the total flow rate of the mixture (without alteration of the flow rate of the noncondensing component) as a result of increase in flow rate of the volatile component. A shift of the maximum towards the outer surface is due to transverse flow of the evaporating component, which slows down the flow of gas mixture at the inner surface of the channel, leading to a shift of the point of maximal v_z towards the outer surface.

An analysis of formula (13) showed that when $c_{12} > c_{11}$ the point τ_{max} is shifted towards the inner cylinder with increase in c_{12}.

The dependence of the longitudinal pressure drop p_l on c_{11} and c_{12} is determined by the variable $\gamma(c_{11}, c_{12})$:

$$p_l(z) = \alpha_1 z = \frac{Q}{\pi n R_2^4}\gamma(c_{11}, c_{12})\, z.$$

The dependence of $\gamma(c_{11}, c_{12})$ on c_{11} when c_{12} = 0 for a vapor–air mixture at temperature 293°K, pressure p_0 = 1 atm, and $\mu = 2\cdot10^{-5}$ kg/m·sec in a channel with R_1/R_2 = 0.5 is shown in Fig. 4, from which it follows that with increase in c_{11} the longitudinal pressure drop in the channel increases and when $c_{11} \to 1$ it tends to infinity.

An analysis of formulas (13), (14) showed that at the limit c_{11}, $c_{12} \to 0$ the formulas for the distributions of v_z and pressure p become the known formulas for a one-component gas

$$v_z = \frac{\alpha_{10}}{4\mu} R_2^2 \left[1 - \tau^2 - \frac{1-(R_1/R_2)^2}{\ln(R_1/R_2)} \ln \tau\right], \quad p = p_0 - \alpha_{10} z,$$

where

$$\alpha_{10} = \frac{8\mu Q \ln(R_1/R_2)}{\pi n R_2^4 \left[1-(R_1/R_2)^2\right] \left\{\left[1+(R_1/R_2)^2\right] \ln(R_1/R_2) + \left[1-(R_1/R_2)^2\right]\right\}}.$$

An isothermal established laminar flow of binary gas mixture in a plane channel was investigated in a similar way to the above-treated case of a coaxial channel. The flow in a plane channel is represented by the system of equations

$$\frac{d}{dx} \rho v_x = 0; \tag{15}$$

$$\rho v_x \frac{dv_x}{dx} = -\frac{\partial p}{\partial x} + \frac{4}{3} \frac{d}{dx}\left(\mu \frac{dv_x}{dx}\right); \tag{16}$$

$$\rho v_x \frac{dv_z}{dx} = -\frac{\partial p}{\partial z} + \frac{d}{dx}\left(\mu \frac{dv_z}{dx}\right); \tag{17}$$

$$\frac{d}{dx} J_{1x} = 0; \tag{18}$$

$$\frac{d}{dx} J_{2x} = 0. \tag{19}$$

System (15), (16) was solved with boundary conditions

$$v_z|_{t=0;1} = 0; \tag{20}$$

$$J_{2x}|_{t=0;1} = 0; \tag{21}$$

$$c_1|_{t=0} = c_{10} = \text{const}; \tag{22}$$

$$c_1|_{t=1} = c_{1h} = \text{const}, \tag{23}$$

where $t = x/h$; x is the transverse coordinate; h is the distance between the plates forming the channel.

The distributions found for v_x, v_z, z, p, c_1, and ρ in this case have their simplest form when μ = const, when they are given by the formulas

$$v_x = \omega\mu/(\rho h); \tag{24}$$

$$v_z = \frac{\alpha_2 h^2}{\mu\omega}\left[t - \frac{1-\exp(\omega t)}{1-\exp\omega}\right]; \tag{25}$$

$$p = p_0 - \alpha_2 z - \frac{\mu^2\omega^2}{\rho h^2}\left\{1 - \frac{\rho}{\rho_h} + \frac{4}{3}\frac{\mu}{D_{12}}\frac{(m_2-m_1)}{m_1\rho}\left[(1-c_{10})\exp(st) - \rho^2(1-c_{1h})/\rho_h^2\right]\right\}; \tag{26}$$

$$c_1 = 1 - (1-c_{10})\exp(st); \tag{27}$$

$$\rho = \{m_1[1-(1-c_{10})\exp(st)] + m_2(1-c_{10})\exp(st)\}n. \tag{28}$$

In formulas (24)-(28)

$$\omega = \frac{nm_1 D_{12}}{\mu} \ln[(1-c_{1h})/(1-c_{10})]; \quad s = \ln[(1-c_{1h})/(1-c_{10})];$$

$$\rho_h = \rho|_{t=1}; \quad p_0 = p|_{t=1;z=0}; \quad \alpha_2 = \frac{Qm_1 D_{12}}{bh^3\psi};$$

$$\psi = \frac{(1-c_{10})}{s^2}\left\{\frac{1}{s} + \frac{1-c_{1h}}{1-c_{10}}\left(1-\frac{1}{s}\right) - \frac{(c_{10}-c_{1h})}{(1-c_{10})\left[1-\left(\frac{1-c_{1h}}{1-c_{10}}\right)^{\omega/s}\right]} + \frac{\left(\frac{1-c_{1h}}{1-c_{10}}\right)^{1+\omega/s} - 1}{(1+\omega/s)\left[1-\left(\frac{1-c_{1h}}{1-c_{10}}\right)^{\omega/s}\right]}\right\}$$

(b is the width of the plates forming the channel).

Equations (15), (18), and (19) show that ρv_x, J_{1x}, and J_{2x} are constants, independent of x. The explanation of the independence of J_{1x} and J_{2x} on x is that in the considered case of steady flow through any plane surface of length l, parallel to the channel generatrices, pass equal flows of molecules of the first (Q_1) and second (Q_2) components of the mixture ($Q_1 = blJ_{1x}$; $Q_2 = blJ_{2x}$). It follows from condition (21) and the constancy of J_{2x} that $J_{2x} = 0$ at any point in the channel. From the expressions for J_{1x} and J_{2x} when $J_{2x} = 0$ we obtain $\rho v_x = m_1 J_{1x}$. Diffusion of molecules of the first and second components occurs in a transverse direction in the presence of a Stefanov flow of gas mixture. In the case of such diffusion J_{2x} will be zero and $\rho v_x = m_1 J_{1x}$, which is consistent with the obtained results.

The equality $\rho v_x = m_1 J_{1x}$ is derived also from formula (24), from which it follows that ρv_x is independent of the longitudinal coordinate.

As an analysis of formula (25) showed, with increase in c_{10} ($c_{10} > c_{1h}$) or with increase in c_{1h} ($c_{1h} > c_{10}$) for a prescribed flow rate Q, there is, as in the case of a coaxial channel, an increase in the maximum value of v_z due to an increase in the total flow of gas mixture and a shift of the point t_{max} (at which v_z is maximal) towards the surface with a lower value of c_1 due to slowing down of the flow of gas mixture at the evaporation surface.

An analysis of (26) showed that with increase in c_{10} ($c_{10} > c_{1h}$) or c_{1h} ($c_{1h} > c_{10}$) the longitudinal pressure drop for a prescribed Q increases.

An analysis of formulas (25), (26) showed that at the limit $c_{10}, c_{1h} \to 0$ the formulas for the distribution of v_z and p become the known formulas for a one-component gas

$$v_z = \frac{\alpha_{20}}{2\mu} h^2 t (1 - t), \quad p = p_0 - \alpha_{20} z,$$

where $\alpha_{20} = 12\mu Q/(bh^3 n)$.

Thus, recondensation of molecules of one of the components of a binary mixture greatly affects the distributions of the longitudinal pressure drop and the longitudinal component of the mass flow velocity in the channel. The formulas obtained in this paper can be used to describe the flow of binary gas mixtures in variable-temperature channels with small temperature drops [1], where the transport coefficients (dynamic viscosity, diffusion coefficient, thermal conductivity) can be regarded as quantities that are independent of temperature.

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CALCULATION OF THE INTERACTION OF A LAMINAR BOUNDARY LAYER WITH AN EXTERNAL SUPERSONIC FLOW BEHIND AN OBSTACLE

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One can cite many papers dealing with investigation of flows in zones of separation and reattachment of a laminar boundary layer [1-12]. In regard to computational methods, it should be noted that the method of interaction of the boundary layer with an external perfect flow, to determine flows in the base region, was first proposed in [1]. However, the lack of sufficient data on the characteristics of the incompressible laminar boundary layer has made it impossible to obtain satisfactory results on base pressure. In [4, 5] the proposed method was modified and applied to the region of interaction of a density shock with a boundary layer [4], and also in the region of separation of the laminar boundary layer on a cylindrical body in transverse flow [5].

The present paper computes flows behind two-dimensional and axisymmetric obstacles, based on a scheme for interaction of the boundary layer with an external perfect flow.

1. We consider the following approximate flow scheme in the base region behind an obstacle washed by a uniform supersonic stream, a scheme of typical interaction of the boundary layer with an external perfect flow (Fig. 1). Between sections 1 and 2 there is flow expansion, AB is a line of constant mass flux, and B is the stagnation point. The broken line denotes the edge of the boundary layer. Immediately behind the body, between sections 2 and 3, there is a constant-pressure separation region, so that the interaction flow begins at some section 3. The calculation of the interaction between the viscous layers and the external, perfect, almost isentropic stream is carried out, as usual, with the boundary layer equations. We write down the system of equations for the compressible laminar boundary layer

$$\frac{\partial(\rho u)}{\partial x} + \frac{\partial(\rho v)}{\partial y} = 0; \tag{1.1}$$

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